

**Week 5 problem solving**  
*+ equations, + applications*

**Drug binding:  $K_d$  vs  $\Delta G$ ,  $\Delta H$ ,  $\Delta S$**   
(in addition to Lecture 9 examples)

**Decay, Radioactivity, gamma-rays**

# $K_d$ calculation from equilibrium concentrations

- **Problem:** A 4  $\mu\text{M}$  target protein solution is made. Then drug is added to achieve 3:5 molar ratio. In equilibrium 2.33  $\mu\text{M}$  of 1:1 complexes are formed. Find the dissociation constant  $K_d$ .
- **Solution:**
  - The molar ratio of 3:5 means that  $[L_{tot}] = 4 \mu\text{M} \frac{3}{5} = 2.4 \mu\text{M}$
  - Since 2.33  $\mu\text{M}$  of complexes was formed, unbound concentrations are:
    - $[P] = [P_{tot}] - [PL] = 4 \mu\text{M} - 2.33 \mu\text{M} = \mathbf{1.67 \mu\text{M}}$
    - $[L] = [L_{tot}] - [PL] = 2.4 \mu\text{M} - 2.33 \mu\text{M} = \mathbf{0.07 \mu\text{M}}$
  - $K_d = [P][L] / [PL] = 1.67 \times 0.07 / 2.33 \approx 0.05 \mu\text{M} = 50 \text{ nM}$

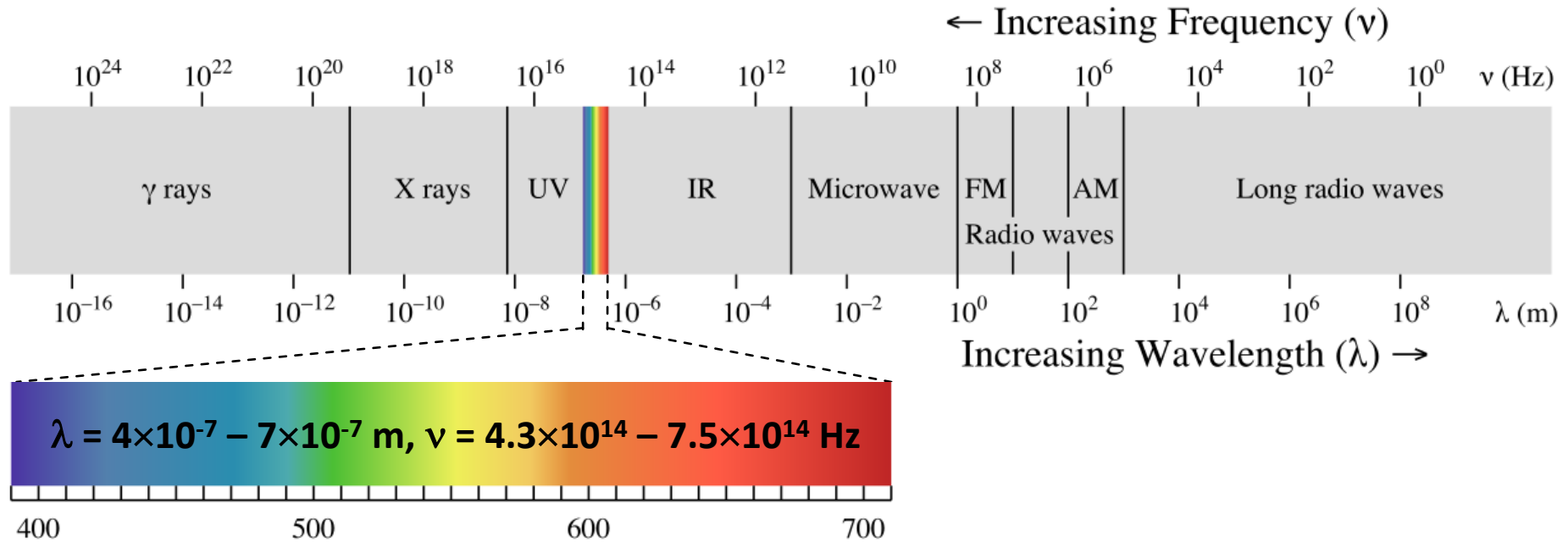
## $K_d$ to $\Delta G$ , $\Delta G = \Delta H - T\Delta S$

- **Problem:** When performed at 300K, an endothermic 1:1 drug/target binding reaction with  $K_d = 58$  nM absorbs the molar heat of 1.4 kcal/mol. What is the molar entropy change of this reaction,  $\Delta S^0$ ? Steps: 1)  $K_d$  to  $\Delta G_0$  2)  $\Delta H_0=1.4$  3)  $\Delta S_0= ..$
- **Solution:**
  - Find molar  $\Delta G_{bind}$ :  $\Delta G = RT \ln Kd = 0.6 \text{ kcal/mol} \times \ln (58 \times 10^{-9}) = -10 \text{ kcal/mol}$
  - Molar  $\Delta H_{bind}$  of +1.4 kcal/mol is given
  - $\Delta G = \Delta H - T\Delta S$ ; therefore,  $\Delta S = (\Delta H - \Delta G) / T = (1.4 + 10 \text{ kcal/mol}) / 300 \text{ K} = 11400 \text{ cal/mol} / 300 \text{ K} = \mathbf{38 \text{ cal/(mol K)}}$
  - In this problem, **entropy change upon binding must be positive!** Given that  $\Delta H$  is positive, this is the only way to achieve negative  $\Delta G$ . It may happen due to hydrophobic nature of binding (the number of water molecules with restricted motion is reduced upon binding).

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**EM radiation,  
fluorescence, radioactivity**

# EM bands & photon energies



Note that energy of ONE photon need to be compared with **kT (not RT)**.

**RT** may be compared with energy of  $N_{\text{Avogadro}}$  of photons.

$$h \approx 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$h \approx 6.6262 \times 10^{-34} \text{ J} \cdot \text{s}$$

(Planck constant)

$$1 \text{ eV} = 1.6022 \times 10^{-19} \text{ J}$$

Energy vs Frequency  
relation:

$$\text{Photon } E = h\nu$$

$$\nu \times \lambda = c$$

$$c \approx 3 \times 10^8 \text{ m/s (speed of light)}$$

## Wavelength, frequency and energy of a single photon

- At room temperature  $kT$ , (energy of one vibration) is equal to

**25.8 meV** (or milli eV)

To break a chemical bond, one needs at least 20  $kT$  of energy

Range	$\lambda$	$\nu$	Photon E, [eV]
Radio	1 m-10 <sup>5</sup> km	3 Hz-300 MHz	12.4 feV-1.24 $\mu$ eV
Microwave	1 mm-1 m	0.3-300 GHz	1.24 $\mu$ eV-1.24 meV
Far IR	15-1000 $\mu$ m	0.3-20 THz	1.24-82.7 meV
IR	1.5-15 $\mu$ m	20-200 THz	82.7-827 meV
Near IR	700-1500 nm	200-430 THz	0.83-1.77 eV
VIS	400-700 nm	430-750 THz	1.77-3.1 eV
Near UV	300-400 nm	0.75-1 PHz	3.1-4.13 eV
UV	200-300 nm	1-1.5 PHz	4.13-6.2 eV
Far UV	10-200 nm	1.5-30 PHz	6.2-124 eV
X-ray	0.01-10 nm	30 PHz-30 EHz	0.13-124 keV
Gamma	<10 pm	>30 EHz	>124 keV

# Gamma, X-ray and visible Light Radiation: wavelength to frequency

- **Problem:** The longest wavelength of EM radiation that is needed to break a C=C bond is 200 nm.
- Calculate the frequency of this radiation in Hz
  - A. 1.5 PHz
  - B. 1.5 THz
  - C. 0.66 GHz
  - D. 60 Hz

- **Solution:**

$$\diamond \nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 10^{-7} \text{ m}} = 1.5 \times 10^{15} \text{ Hz} = 1.5 \text{ PHz}$$

T is Tera (12)  
P is Peta (15)

- **Answer:** 1.5 PHz

# Ultraviolet (UV) radiation: from frequency to photon energy

- The longest wavelength of UV radiation that is needed to break a C=C bond is 200 nm
- Calculate the frequency of this radiation in Hz
- Calculate the energy of a single photon in eV
  - A. 47 meV
  - B. 6.2 eV
  - C. 3.1 keV
  - D. 9.3 MeV
- **Solution:**
  - Have just found that frequency  $\nu = 1.5 \text{ PHz}$
  - $E_{\text{photon}} = h\nu = (4.1357 \times 10^{-15} \text{ eV}\cdot\text{s}) \times (1.5 \times 10^{15} \text{ Hz}) = 6.2 \text{ eV}$
- **Answer:** 6.2 eV. , keep in mind that thermal energy at room temperature  $kT \sim 0.0258 \text{ eV}$



# Activity of radiopharmaceuticals

- Counting the *rate of radionuclei disintegration*
- 1 Bq (Becquerel) = 1 event per second =  $1 \text{ s}^{-1}$
- 1 Ci (Curie) = 37 GBq =  $37 \cdot 10^9 \text{ Bq}$
- Energy released depends on the specific nuclei and disintegration mode
  - an average value may be associated with each radionuclide and decay mode
- Activity goes down as the radionuclide decays
  - provided values are “*at time of calibration*”
- Radiopharmaceuticals are prescribed as *activity* in Bq**

<b>Sodium Iodide (<sup>131</sup>I) Solution</b>  <b>Therapeutic Oral</b>  USES: Compounding of Oral Therapeutic Solution or Capsules  Each ml contains: Carrier-free Na <sup>131</sup> I Inactive Ingredients: 0.05M Sodium Hydroxide and 0.02M Sodium Thiosulfate Store upright in a shielded container at 2° - 25° C Calculate dosage from Calibration date  <b>RX Only</b>  <sup>131</sup> I HALF LIFE= 8.025 DAYS  	Order #	Lot #:
	Calibration Date: Calibration Time: 12:00 (MST) Total radioactivity @ calibration: GBq (mCi) Total Volume @ calibration: ml Radioconcentration @ calibration: GBq/ml (mCi/ml)  Expiration Date: NDC Code:	<b>Caution-Radioactive Material</b>  Manufactured by International Isotopes, Inc. Idaho Falls, ID

**Table 2. Solution Strengths**

Total Radioactivity* per Vial		
Concentration*	Volume of Solution	Total Radioactivity* per Vial
185 MBq/mL (5 mCi/mL)	1 mL	185 MBq (5 mCi)
	2 mL	370 MBq (10 mCi)
	3 mL	555 MBq (15 mCi)
	4 mL	740 MBq (20 mCi)
	5 mL	925 MBq (25 mCi)
925 MBq/mL (25 mCi/mL)	7 mL	1295 MBq (35 mCi)
	2 mL	1850 MBq (50 mCi)
	3 mL	2775 MBq (75 mCi)
	4 mL	3700 MBq (100 mCi)
	6 mL	5550 MBq (150 mCi)

Source: FDA \* *At time of calibration*

# Dosimetry-related quantities

- **Radiation source, Exposure:** a measure of ionization  
Electric charge freed by radiation per **kg of air**  
 $1 \text{ R (Roentgen)} = 0.000258 \text{ C/kg (Coulomb per kg)}$
- **Absorbed dose (energy/body mass):**
  - ❖  $1 \text{ Gy (gray)} = 1 \text{ J/kg}$
  - ❖  $1 \text{ rad} = 0.01 \text{ Gy}$
  - ❖ Depends on the type of matter that absorbs the radiation, e.g. for an exposure of 1 roentgen by 1 MeV  $\gamma$ -rays:
    - the dose in air = 0.877 rad
    - the dose in water = 0.975 rad
    - the dose in averaged human tissue = 0.965 rad
- **Dose equivalent:**
  - ❖ Different radiation types have different biological effects for the same deposited energy
  - ❖  $W_R$  is a **corrective radiation weighting factor**:
    - dependent on radiation type
    - converts the absorbed dose into an estimate of tissue damage
  - ❖  $1 \text{ Sv (Sievert)} = W_R \times \text{Gy} = W_R \times [\text{J/kg}]$
  - ❖  $1 \text{ rem (roentgen equivalent man)} = 0.01 \text{ Sv}$
- **Ionizing radiation is prescribed as *absorbed dose in Gy***

# Activity of radio-pharmaceuticals

- **Problem:**  $^{131}\text{I}$  decay releases energy in the form of  $\beta$  and  $\gamma$  radiation. The average energy release per nucleus is **192 keV for  $\beta$**  and 364 keV for  $\gamma$ .

Given a sample of  $^{131}\text{I}$  with total activity of 5 mCi, how much energy does it emit per second in the form of  $\beta$ -radiation? Give your answer in J/s.

- A. 57 kJ/s
  - B. 28.4 J/s
  - C. 13.2 mJ/s
  - D. 5.68  $\mu\text{J/s}$
- **Solution:**
    - 1 Bq = 1 nucleus per second =  $1 \text{ s}^{-1}$ ; 1 Ci = 37 GBq
    - 5 mCi =  $0.005 \times 37 \text{ GBq} = 185 \text{ MBq}$  ( $1.85 \times 10^8$  nuclei disintegrate per second) : converting Curie units to Bq units.
    - For  $\beta$ :  $(1.92 \times 10^5 \text{ eV}) \times (1.85 \times 10^8 \text{ Bq/s}) \times (1.6 \times 10^{-19} \text{ J/eV}) = 5.68 \times 10^{-6} \text{ J/s}$   
**Beta-energy** x **Number of decays** x **eV to J conversion**
  - **Answer:** 5.68  $\mu\text{J/s}$