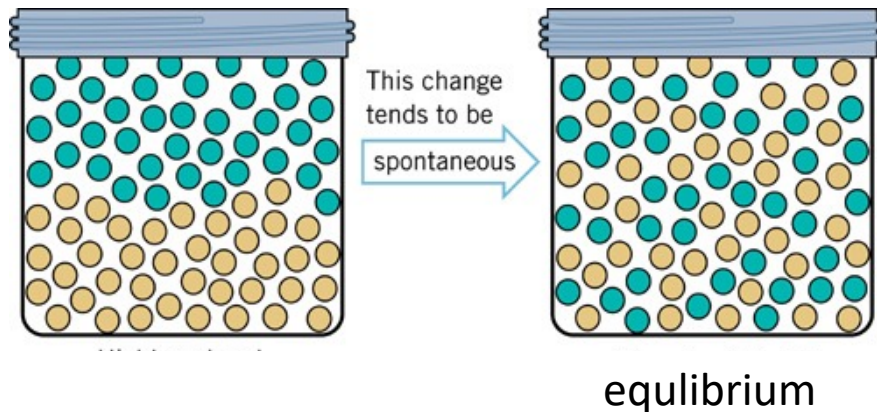


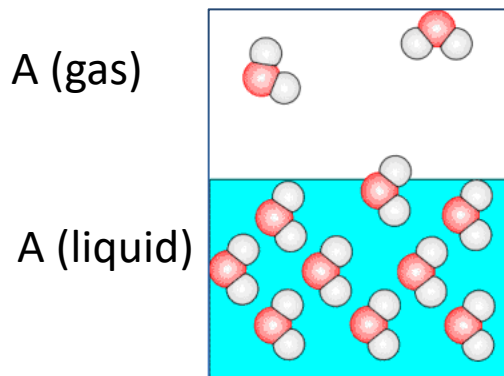
Mixtures, Equilibrium & Chemical Potential μ

From minimizing Gibbs Free Energy to the Equilibrium Constant (K)



Single compartment

- Total G \rightarrow minimum
- **Solutions**
- Suspensions
- Colloids (milk, nanoparticles, colloidal silver, sulphur, gold)



Two compartments

- Chemical potential μ_A of *each component is equal* in both compartments in equilibrium

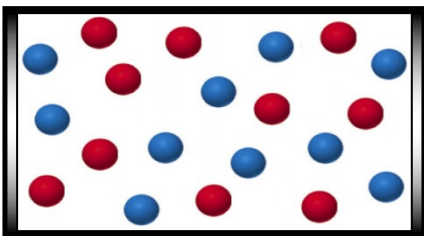
Mixtures: Chemical Potential μ

- $G = H - TS$ is an extensive variable
- For a **pure substance J** the chemical potential is defined as molar Gibb's energy: $\mu_J \equiv G_m = G/n$
- **Chemical Potential is molar Free Energy of one chemical ingredient in a mixture**
- In a mixture of n_1, n_2, \dots, n_J
 - n_j is the number of moles of compound j

$$\mu_J = \left(\frac{\partial G}{\partial n_J} \right)_{p, T, n_{i \neq J}}$$



The chemical potential in solution



Chemical potential (free energy per particle i) in solution increases with **log**(concentration) for entropic reasons (less states):

$$\mu_i^{gas} = \mu_i^{P_0} + RT \ln \frac{P_i}{P_0}$$
$$\mu_i = \mu_i^{c_0} + RT \ln \frac{c_i}{c_0}$$

$$\mu_i = \mu_i^{c_0=1} + RT \ln c_i$$

We may choose units and standard states, eg 1 M

For non-ideal solutions: c_i becomes effective concentration a_i

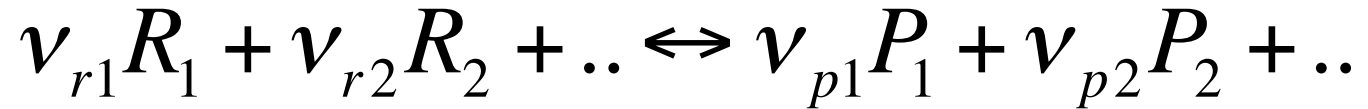
The chemical **activity** a_i , an *effective* particle concentration, replaces c_i for non-ideal solutions: $a_i = \gamma_i \cdot c_i$

- $a_i \equiv$ activity of species i ;
- $\gamma_i \equiv$ activity coefficient, $\gamma_i = 1$, $a_i = c_i$ for ideal solution;
- $c_i =$ concentration of species i , c_0 is 'standard or reference' concentrations for which the chemical potential is known

Equilibrium Concentrations vs ΔG^0

Reactants

Products



- **Reaction** (ν 's are stoichiometric coefficients)
- **Equilibrium constant**
- **Reaction Free Energy**
- **Equilibrium concentrations are related to ΔG^0**

$$K \equiv \frac{C_{product1}^{\nu_{p1}} C_{product2}^{\nu_{p2}} \dots}{C_{react1}^{\nu_{r1}} C_{react2}^{\nu_{r2}} \dots}$$

$$\Delta G^0 \equiv \sum_{products} \nu_{pi} \mu_i^0 - \sum_{reactants} \nu_{rj} \mu_j^0$$

$$\ln K = -\frac{\Delta G^0_{reaction}}{RT}$$

The Equilibrium Constant, K and concentration ratios. Examples

For reaction: $A \leftrightarrow B$: $K = C_B / C_A$

For reaction: $aA + bB \leftrightarrow cC + dD$

the equilibrium constant is written as: $K = \frac{C_C^c C_D^d}{C_A^a C_B^b}$

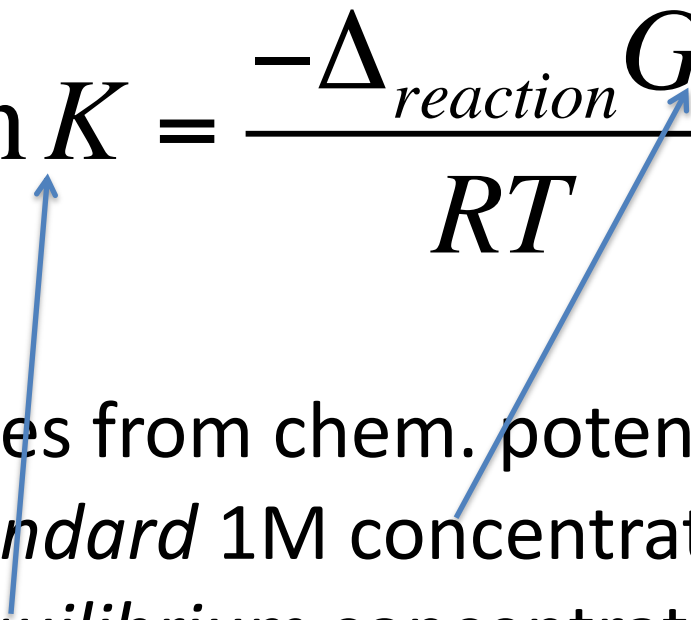
- **Units** of K depend on the reaction stoichiometry,
- Examples:

- $A \leftrightarrow B$: $K = C_B / C_A$ K is dimensionless
- $D + R \leftrightarrow DR$: $K = C_{DR} / (C_D C_R)$ K [M^{-1}], [M] for mole/liter
- $DR \leftrightarrow D + R$: $K = (C_D C_R) / C_{DR}$ K [M]

Review: concentrations and energies

Relationship between Gibbs free energy of reaction and the equilibrium concentrations is defined by constant K

$$\Delta_{reaction} G^{\circ} = \Delta H^{\circ} - T\Delta S^{\circ}$$

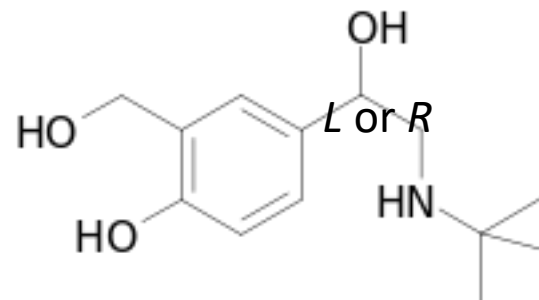
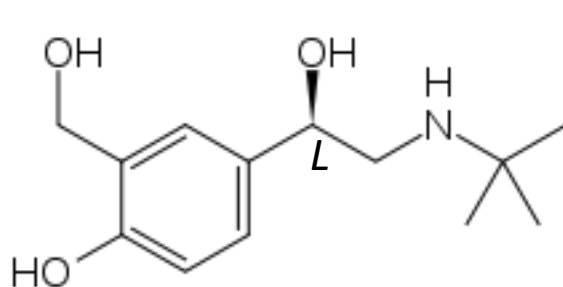
$$\ln K = \frac{-\Delta_{reaction} G^{\circ}}{RT}$$


Where G° comes from chem. potentials (molar free energies) at *standard* 1M concentrations, while K is calculated at *equilibrium* concentrations.

Example Reaction: Chiral Transition

$$\ln K = \frac{-\Delta_{\text{reaction}} G^{\circ}}{RT}$$

- ΔG° contains Enthalpic (ΔH°) and Entropic ($-T\Delta S^{\circ}$) contributions. How do we measure them?
- Example: **Levosalbutamol** vs **Salbutamol** (racemic)



- Chiral transition $L \leftrightarrow R$: $\ln(C_R/C_L) = -(G_R - G_L)/(RT)$

“As a bronchodilator, it is used to treat asthma and Chronic obstructive pulmonary disease (COPD). In general, levosalbutamol has similar pharmacokinetic and pharmacodynamic properties to salbutamol; however, its manufacturer, Sepracor, has implied (although not directly claimed) that the presence of only the R-enantiomer produces fewer side effects.” Wikipedia

Finding Enthalpy and Entropy from K at several temperatures: *van't Hoff Equation*

Alternative to Calorimetry to determine ΔG° . Idea:

- Equilibrium concentrations and K depend on T
- Higher T will favor the reaction direction with gain of entropy.
- Recipe: Measure K (T) and derive ΔG° from that dependence

Substituting ΔG for $\Delta H - T\Delta S$:

$$\ln K = -\frac{\Delta_r H^\circ}{RT} + \frac{\Delta_r S^\circ}{R}$$

Equilibrium constant at two different temperatures:

$$\ln K_1 = \frac{-\Delta_r H^\circ}{RT_1} + \frac{\Delta_r S^\circ}{R}$$

$$\ln K_2 = \frac{-\Delta_r H^\circ}{RT_2} + \frac{\Delta_r S^\circ}{R}$$

Subtracting:

$$\ln\left(\frac{K_1}{K_2}\right) = \frac{\Delta_r H^\circ}{R} \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$



Jacobus Henricus
van't Hoff (1852-1911),
a Dutch chemist

Plotting $\ln K$ vs $1/T$ to derive ΔH° and ΔS° of a reaction or transition

$$\ln K = -\frac{\Delta_r H^\circ}{RT} + \frac{\Delta_r S^\circ}{R}$$

The slope is $-\Delta_r H^\circ / R$

The intercept is $\Delta_r S^\circ / R$

Van't Hoff Equation with \log_{10}

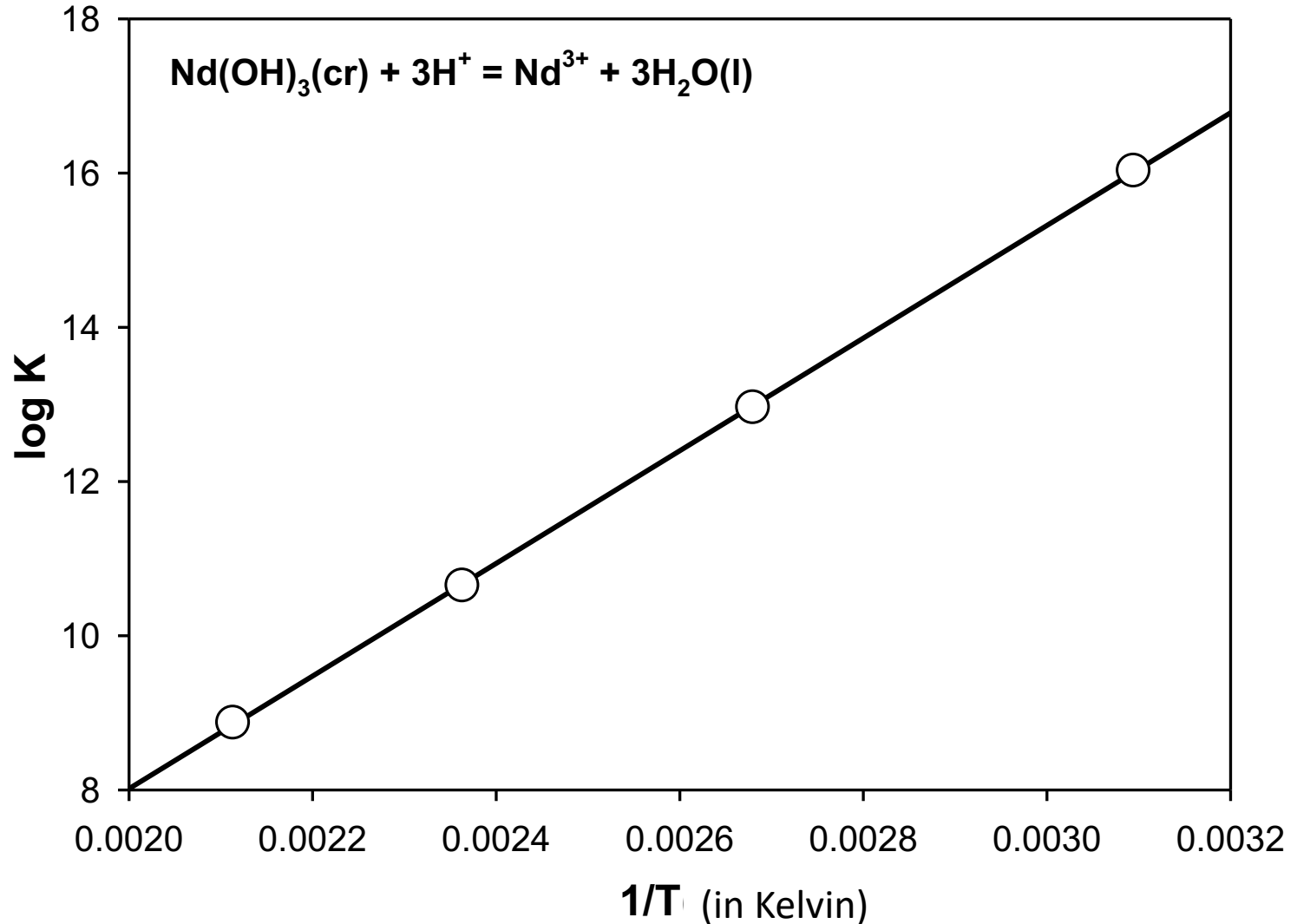
$$\log_{10} K = -\frac{\Delta_r H^\circ}{2.3RT} + \frac{\Delta_r S^\circ}{2.3R}, \quad 2.3 = \ln(10)$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

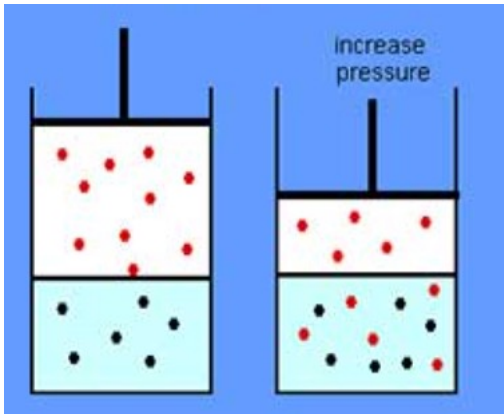
$$\log_{10}(x) = \ln(x)/\ln(10)$$

$$\ln(x) = 2.3 \log_{10}(x)$$

Example of van't Hoff Plot



Partial Pressures in Gas mixtures vs Solution Concentrations



Equilibrium between mixtures in gas and liquid:

- Concentration of an ingredient in solution is proportional to its partial pressure in the gas phase
- Coefficients depend on the solvent (e.g. water) and ingredient-solvent interactions

$$C_{i, \text{ in solution}} = K_i P_{i, \text{ in gas}}$$

It can be derived from the condition that chemical potential for of molecule i is equal in gas and liquid phases.

Gas-Solution Equilibrium for Each Solution Ingredient

$$\mu_{i,liq} = \mu_{i,vap} \quad ; \quad \mu_{i,liq}^* = \mu_{i,vap}^*$$

$$\mu_{i,liq}^* + RT \ln\left(\frac{c_i}{c_i^*}\right) = \mu_{i,vap}^* + RT \ln\left(\frac{p_i}{p_i^*}\right)$$

$$RT \ln\left(\frac{c_i}{c_i^*}\right) = RT \ln\left(\frac{p_i}{p_i^*}\right)$$

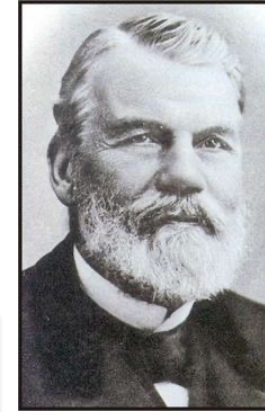
$$\left(\frac{c_i}{c_i^*}\right)_i = \left(\frac{p_i}{p_i^*}\right)$$

C is Concentration in liquid p is Partial pressure in gas

Conc

- i may be water or drug
- x_i - molar fraction of solute or solvent
- * - ingredient at standard concentration or pressure

Raoult's Law: solvent vapor pressure for solutions



French physicist
François-Marie Raoult

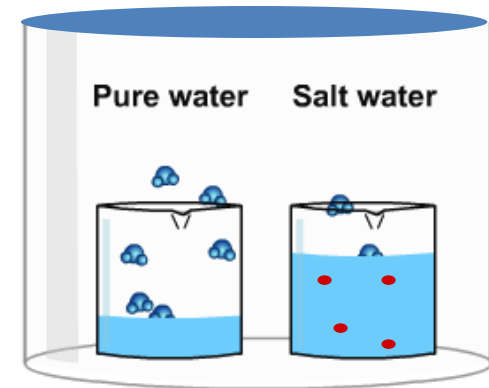
$$x_i^w \equiv n_i^w / n_{total}^w = p_i^g / p_i^{*g pure}$$

i for solvent (e.g. water) molecules

- **Solvent** (eg water) **pressure** vs molar fraction x_{solute} of *non-volatile solute*
- Vapor pressure of *solvent* is decreased as the *solute* concentration is increased
- $P_{water}^* = 0.23 \text{ bar at } 20^\circ\text{C}$ (100C?)

$$P_w = x_w P_w^* = (1 - x_{solute}) P_w^*$$

$$P_w^* - P = \Delta P = x_{solute} P_w^*$$



- Water pressure will be lower as you add salt
- Salty water will boil at higher temperature

Henry's Law (gas in solvent)



William Henry,
a British chemist

$$x_i^{liq} = p_i^g / p_i^{*g\ pure}$$

i is gas component (eg oxygen)

- Gas dissolves in liquid proportionally to its pressure. Example: Oxygen in blood

$$P_{i,gas} = x_{i,sol} \cdot K_H^x$$

Henry's Law

- Here K is an empirical constant, slope of the tangent to the experimental curve.
- In the gas mixture itself, **partial pressures are proportional to molar fractions**: $P_i = x_i P_0$ - *Dalton's law*

Mixtures that obey

Henry's Law for $x_{solute} \rightarrow 0$ and Raoult's Law for $x_{solvent} \rightarrow 1$

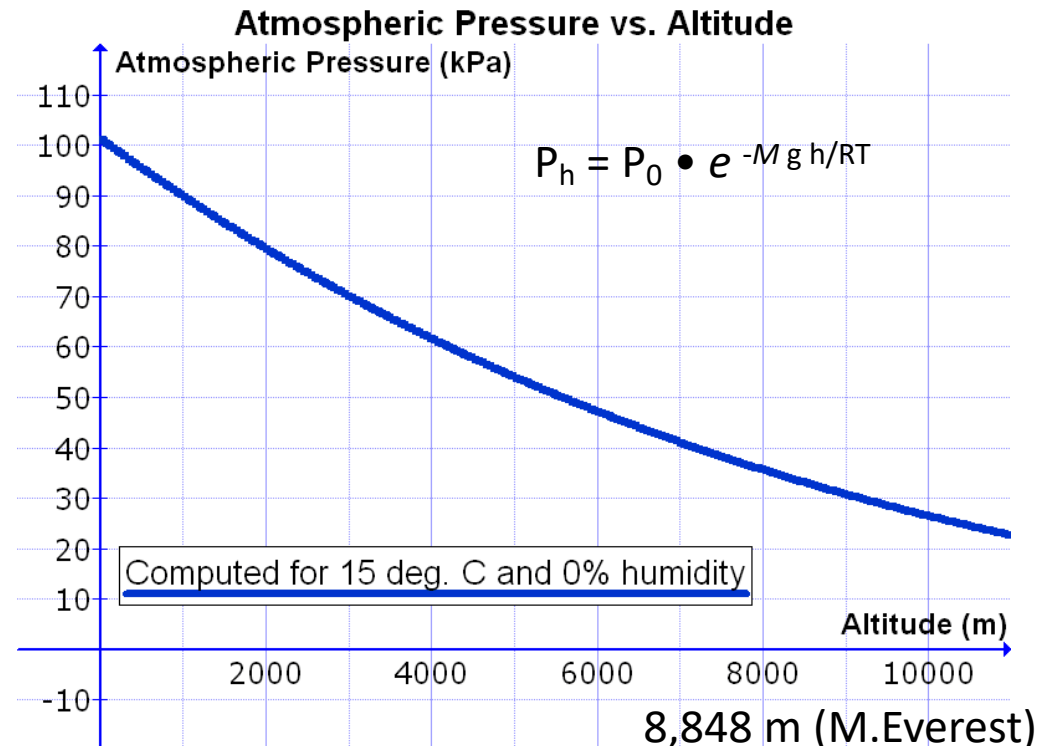
are called *ideal-dilute solutions*.

Air Pressure , O₂ in Blood

- Rearranged Henry's law: $[C_{i,aq}] = P_i / K_{Henry}$
- oxygen (O₂) : $K_H=769.2 \text{ L}\cdot\text{atm}/\text{mol}$, $[O_2] \sim 270 \mu\text{M}$
- carbon dioxide (CO₂) : $K_H=29.4 \text{ L}\cdot\text{atm}/\text{mol}$, $[CO_2]=13.2\mu\text{M}$
- hydrogen (H₂) : $K_H=1282.1 \text{ L}\cdot\text{atm}/\text{mol}$

Total pressure P and partial pressures P_i also depends on elevation (depth) and temperature.

Solubility of O₂ in plasma also depends on hemoglobin (Hb) that binds and carries O₂



Review

- The chemical potential of component J:
 - Gas
 - Liquid mixture
 - ΔG and entropy of mixing.
- The chemical equilibrium
 - K via concentrations and reaction stoichiometry
 - From K, to ΔG°
 - From K at T_1 and T_2 , to ΔH° and ΔS° , Van't Hoff

$$\mu_i^g = \mu_i^{P_0} + RT \ln \frac{P_i}{P_0}$$

$$\mu_i = \mu_i^{c_0} + RT \ln \frac{c_i}{c_0}$$

a_i below may also be molar fraction x_i or concentration c_i depending on the standard state and ideality

$$K = \prod_{i=1}^n a_i^{v_i}, \quad \text{e.g. } K = \frac{[A \bullet B]}{[A][B]}$$

$$\ln K = -\frac{\Delta_r G^\circ}{RT}$$

$$\ln K = -\frac{\Delta_r H^\circ}{RT} + \frac{\Delta_r S^\circ}{R} \quad \ln\left(\frac{K_1}{K_2}\right) = -\frac{\Delta H^\circ}{R} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$